Riviera International Academy

Assignment-2077

(Bhadra 11, 2077, Thursday)

Class: Eight

Subject- Mathematics

Source: Photos of exercise are given below.
Work: Read & write concept of Laws of indices.
Do your work neatly

Unit 9 Laws of Indices

9.1 Laws of Indices - Review

Let's consider an algebraic term $4x^3$.

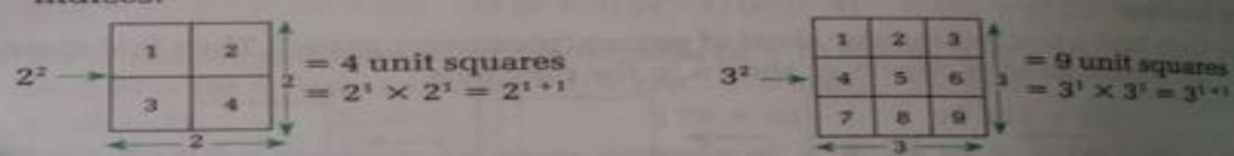
Here, 4 is the coefficient, x is the base and 3 is the exponent of the base. The expon[ent] of a base is also called its index (plural: indices).

Power is an expression that represents repeated multiplication of the same number [or] variable) whereas exponent refers to a quantity that represents the power to which th[e] number (or variable) is raised. Both terms are often used interchangeably in mathemati[cal] operations.

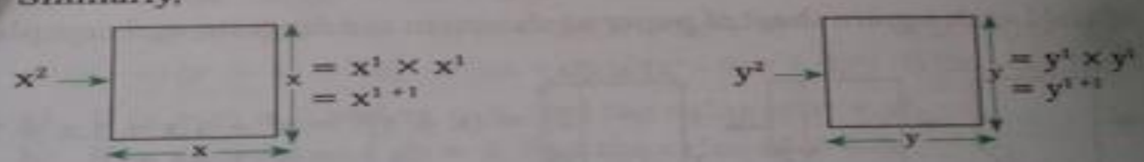
There are certain rules for the operations of indices of different algebraic terms. The[se] rules are also called the **Laws of indices.**

(i) Product law of indices

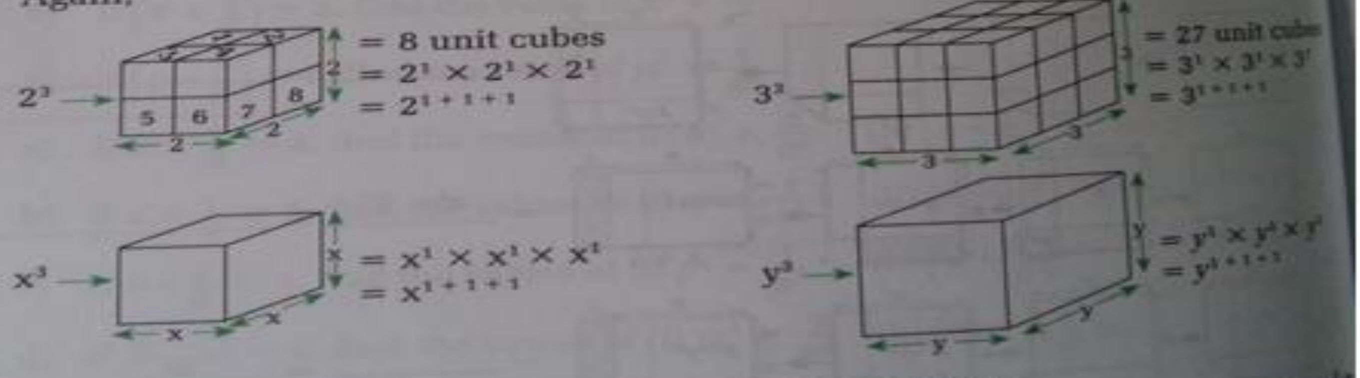
Study the following illustrations and investigate the idea of the product law o[f] indices.



Similarly,



Again,



Also, $2^2 \times 2 = 2^{2+1} = 2^3$, $3^2 \times 3^3 = 3^{2+3} = 3^5$, and so on.
Thus, if a^m and a^n are any two terms with the same base a and the powers m and n respectively, then, $a^m \times a^n = a^{m+n}$

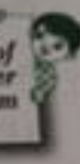
(ii) Quotient law of indices

Let's study the following illustrations and investigate the idea of the quotient law of indices.

$2^2 \div 2 = \frac{2^2}{2} = \frac{2 \times 2}{2} = 2 = 2^{2-1}$

$2^4 \div 2^2 = \frac{2^4}{2^2} = \frac{2 \times 2 \times 2 \times 2}{2 \times 2} = 2^2 = 2^{4-2}$

$2^3 \div 2^5 = \frac{2^3}{2^5} = \frac{2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2} = \frac{1}{2^2} = \frac{1}{2^{5-3}}$



It's easier!
In the case of division of the same bases, smaller power is subtracted from the greater power!!

Thus, if a^m and a^n are any two terms with the same base a and powers m and n respectively, then,

$$a^m \div a^n = a^{m-n} \text{ if } m > n \text{ and } a^m \div a^n = \frac{1}{a^{n-m}} \text{ if } m < n$$

(iii) Power law of indices

Study the following illustrations and investigate the idea of the power law of indices.

$(2^2)^2 = 2^2 \times 2^2 = 2^4 = 2^{2 \times 2}$

$(2^3)^2 = 2^3 \times 2^3 = 2^6 = 2^{3 \times 2}$

$(2^4)^3 = 2^4 \times 2^4 \times 2^4 = 2^{12} = 2^{4 \times 3}$

I got it!
$(x^2)^3 = x^{2 \times 3} = x^6$
$(y^3)^4 = y^{3 \times 4} = y^{12}$

Thus, if a^m is any term with the base a and the index m, then, $(a^m)^n = a^{m \times n}$

Furthermore, if a and b are any two terms, then,

$$(a \times b)^m = a^m \times b^m \quad \text{and} \quad \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

(iv) Law of negative index

If a^{-m} is a term with base a and power $-m$, then,

$$a^{-m} = \frac{1}{a^m} \quad \text{or, } \frac{1}{a^m} = a^{-m} \quad \text{or} \quad a^m = \frac{1}{a^{-m}}$$

For example,

$2^{-2} = \frac{1}{2^2}$, $\frac{1}{2^4} = 2^{-4}$, $3^4 = \frac{1}{3^{-4}}$

Verification

$\frac{x^3}{x^5} = \frac{x \times x \times x}{x \times x \times x \times x \times x} = \frac{1}{x^2}$

From the quotient law,

$\frac{x^3}{x^5} = x^{3-5} = x^{-2}$

So, $\frac{1}{x^2} = x^{-2}$

(v) Law of zero index

Study the following illustrations and investigate the idea of the law of zero index.

$2^0 = 2^{1-1} = 2^1 \div 2^1 = \frac{2^1}{2^1} = 1$

$3^0 = 3^{1-1} = 3^1 \div 3^1 = \frac{3^1}{3^1} = 1$

Thus, if a^0 is any term with base a and power 0, then, $a^0 = 1$

(vi) Root law of indices

$\sqrt[2]{\ }$ (or only $\sqrt{\ }$) is the 2^{nd} order of root, $\sqrt[3]{\ }$ is the 3^{rd} order of root, and so on.

Therefore, $\sqrt[n]{\ }$ is the n^{th} order of root.

Again, the square root of $4 = \sqrt{4} = \sqrt{2^2}$ or $\sqrt[2]{2^2} = 2^{\frac{2}{2}} = 2$

The square root of $16 = \sqrt{16} = \sqrt{2^4}$ or $\sqrt[2]{2^4} = 2^{\frac{4}{2}} = 2^2 = 4$

The cube root of $27 = 3^3 = \sqrt[3]{3^3} = 3^{\frac{3}{3}} = 3$

In this way, $\sqrt[2]{2^2} = 2^{\frac{2}{2}}$, $\sqrt[2]{2^4} = 2^{\frac{4}{2}}$, $\sqrt[3]{3^3} = 3^{\frac{3}{3}}$... and so on.

In the similar way, $\sqrt[n]{a^m} = a^{\frac{m}{n}}$.

Thus, if $a^{\frac{m}{n}}$ is a term with base a and power $\frac{m}{n}$, then, $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

Laws of indices at a glance

(i)	Product law	$a^m \times a^n = a^{m+n}$
(ii)	Quotient law	$a^m \div a^n = a^{m-n}$ when $m > n$ $a^m \div a^n = \frac{1}{a^{n-m}}$ when $m < n$
(iii)	Power law	$(a^m)^n = a^{m \times n}$, $(ab)^m = a^m b^m$, $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
(iv)	Law of negative index	$a^{-m} = \frac{1}{a^m}$ or $a^m = \frac{1}{a^{-m}}$
(v)	Law of zero index	$a^0 = 1, b^0 = 1, x^0 = 1$ and so on
(vi)	Root law of index	$a^{\frac{m}{n}} = \sqrt[n]{a^m}$ or $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

Worked-out examples

Example 1: ***Find the products in their exponential forms.***

(i) $5 \times 5^2 \times 5^3 \times 5^5$ *(ii)* $3^2 \times 9^2 \times 27^2$

(iii) $(2a)^4 \times (2a)^{-12} \times (2a)^2$

Solution:

(i) $5 \times 5^2 \times 5^3 \times 5^5 = 5^{1+2+3+5}$ — In product law, $a^m \times a^n = a^{m+n}$

$= 5^{11}$

(ii) $3^2 \times 9^2 \times 27^2 = 3^2 \times (3^2)^2 \times (3^3)^2$

$= 3^2 \times 3^{2 \times 2} \times 3^{3 \times 2}$ — In power law, $(a^m)^n = a^{m \times n}$

$= 3^2 \times 3^4 \times 3^6 = 3^{12}$

(iii) $(2a)^4 \times (2a)^{-12} \times (2a)^2 = (2a)^{4-12+2}$

$= (2a)^{-6} = \frac{1}{(2a)^6}$ — Law of negative index: $a^{-m} = \frac{1}{a^m}$

Example 2: ***Find the quotient in exponential forms.***

(i) $(7^3)^3 \div 7^8$ *(ii)* $16^4 \div 8^3$ *(iii)* $27^2 \div 9^6$

Solution:

(i) $(7^3)^5 \div 7^8 = 7^{15} \div 7^8$
$= 7^{15-8} = 7^7$

In quotient law of indices, $a^m \div a^n = a^{m-n}$ if $m > n$

(ii) $16^4 \div 8^2 = (2^4)^4 \div (2^3)^2$
$= 2^{16} \div 2^6$
$= 2^{16-6} = 2^{10}$

(iii) $27^2 \div 9^6 = (3^3)^2 \div (3^2)^6$
$= 3^6 \div 3^{12}$
$= \frac{1}{3^{12-6}} = \frac{1}{3^6}$

In quotient law of indices, $a^m \div a^n = \frac{1}{a^{n-m}}$ if $m < n$

Example 3: ***Evaluate*** *(i)* $\left(\frac{8}{27}\right)^{\frac{2}{3}}$ *(ii)* $\left(\frac{25}{36}\right)^{0.5}$ *(iii)* $\left(\frac{125}{64}\right)^{-\frac{1}{3}}$

Solution:

(i) $\left(\frac{8}{27}\right)^{\frac{2}{3}} = \left(\frac{2^3}{3^3}\right)^{\frac{2}{3}} = \left(\frac{2}{3}\right)^{3\times\frac{2}{3}} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

(ii) $\left(\frac{25}{36}\right)^{0.5} = \left(\frac{25}{36}\right)^{\frac{1}{2}} = \left(\frac{5^2}{6^2}\right)^{\frac{1}{2}} = \left(\frac{5}{6}\right)^{2\times\frac{1}{2}} = \frac{5}{6}$

(iii) $\left(\frac{125}{64}\right)^{-\frac{1}{3}} = \left(\frac{64}{125}\right)^{\frac{1}{3}}$

$\left(\frac{125}{64}\right)^{-\frac{1}{3}} = \frac{125^{-\frac{1}{3}}}{64^{-\frac{1}{3}}} = \frac{\frac{1}{125^{\frac{1}{3}}}}{\frac{1}{64^{\frac{1}{3}}}} = \frac{1}{125^{\frac{1}{3}}} \times \frac{64^{\frac{1}{3}}}{1} = \frac{64^{\frac{1}{3}}}{125^{\frac{1}{3}}} = \left(\frac{64}{125}\right)^{\frac{1}{3}}$

$= \left(\frac{4^3}{5^3}\right)^{\frac{1}{3}}$

$= \left(\frac{4}{5}\right)^{3\times\frac{1}{3}} = \frac{4}{5}$

Example 4: ***Simplify*** *(i)* $\sqrt[3]{125x^6y^3}$ *(ii)* $\sqrt[3]{\sqrt{64}}$

Solution:

(i) $\sqrt[3]{125x^6y^3} = \sqrt[3]{5^3x^6y^3} = 5^{\frac{3}{3}}\,x^{\frac{6}{3}}\,y^{\frac{3}{3}} = 5x^2y$

Root law of indices: $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

(ii) $\sqrt[3]{\sqrt{64}} = \sqrt[3]{8} = \sqrt[3]{2^3} = 2^{\frac{3}{3}} = 2$

Example 5: ***Simplify*** *(i)* $\frac{8^3 \times 15^3}{6^4 \times 10^3}$ *(ii)* $(x^{a+b})^{a-b} \times (x^{b+c})^{b-c} \times (x^{c-a})^{c+a}$

Solution:

(i) $\frac{8^3 \times 15^3}{6^4 \times 10^3} = \frac{(2^3)^3 \times (3\times 5)^3}{(2\times 3)^4 \times (2\times 5)^3}$

$= \frac{2^9 \times 3^3 \times 5^3}{2^4 \times 3^4 \times 2^3 \times 5^3} = \frac{2^9 \times 3^3 \times 5^3}{2^7 \times 3^4 \times 5^3} = \frac{2^{9-7} \times 5^{3-3}}{3^{4-3}} = \frac{2^2 \times 5^0}{3} = \frac{4}{3}$

Subject- Opt. Mathematics

Learn transformation statistic for oral test.

Subject-Extra Reading

Subject- Science

1. Write any four advantages of modern periodic table.
2. Write three differences between modern and Mendeleev periodic table in three points.
3. What is group of periodic table? Write the differences between group and period in two points.
4. Which group of modern periodic table is called alkali earth metal and why? Why inert gases are kept separately in Modern periodic table.

The End.